

## Yukawa Unification, $b \rightarrow s\gamma$ and Bino-Stau Coannihilation

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### Abstract

The minimal supersymmetric standard model with universal boundary conditions and “asymptotic” Yukawa unification is considered. The full one-loop effective potential for radiative electroweak symmetry breaking as well as the one-loop corrections to the charged Higgs boson,  $b$ -quark and  $\tau$ -lepton masses are included. The CP-even Higgs boson masses are corrected to two-loops. The relic abundance of the lightest supersymmetric particle (bino) is calculated by including its coannihilations with the next-to-lightest supersymmetric particle (lightest stau) consistently with Yukawa unification. The branching ratio of  $b \rightarrow s\gamma$  is evaluated by incorporating all the applicable next-to-leading order QCD corrections. The bino-stau coannihilations reduce the bino relic abundance below the upper bound from cold dark matter considerations in a sizable fraction of the parameter space allowed by  $b \rightarrow s\gamma$  for  $\mu > 0$ . Thus, the  $\mu > 0$  case, which also predicts an acceptable  $b$ -quark mass, is perfectly compatible with data.

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It is well-known [1] that the assumption that all three Yukawa couplings of the third family of quarks and leptons unify “asymptotically” (i.e., at the grand unified theory (GUT) mass scale  $M_{GUT} \sim 10^{16}$  GeV) naturally restricts the top quark mass to large values compatible with the present experimental data. Such a Yukawa unification can be obtained by embedding the minimal supersymmetric standard model (MSSM) in a supersymmetric (SUSY) GUT with a gauge group such as  $SU(4)_c \times SU(2)_L \times SU(2)_R$ ,  $SO(10)$  or  $E_6$  which contain  $SU(4)_c$  and  $SU(2)_R$ . Assuming that the electroweak Higgs superfields  $H_1, H_2$  and the third family right-handed quark superfields  $t^c, b^c$  form  $SU(2)_R$  doublets, we obtain [2] the “asymptotic” Yukawa coupling relation  $h_t = h_b$  and, hence, large  $\tan \beta \approx m_t/m_b$ . Moreover, if the third generation quark and lepton  $SU(2)_L$  doublets (singlets)  $Q_3$  and  $L_3$  ( $b^c$  and  $\tau^c$ ) form a  $SU(4)_c$  4-plet ( $\bar{4}$ -plet) and the electroweak Higgs  $H_1$  which couples to them is a  $SU(4)_c$  singlet, we obtain  $h_\tau = h_b$  and the successful “asymptotic” mass relation  $m_\tau = m_b$  follows.

The simplest and most restrictive version of MSSM with gauge coupling unification is based on the assumption of radiative electroweak symmetry breaking with universal boundary conditions from gravity-mediated soft SUSY breaking. The tantalizing question is then whether this scheme is compatible with exact “asymptotic” unification of the three third family Yukawa couplings. A positive answer to this question would be very desirable since it would lead to a simple and highly predictive theory. This issue has been systematically studied in Ref. [3].

A significant problem, which may be faced in trying to reconcile Yukawa unification and universal boundary conditions, is due to the generation of sizeable SUSY corrections to the  $b$ -quark mass [3,4]. The sign of these corrections is opposite to the sign of the MSSM parameter  $\mu$  (with the conventions of Ref. [5]). As a consequence, for  $\mu < 0$ , the tree-level value of  $m_b$ , which is predicted from Yukawa unification already near its experimental upper bound, receives large positive corrections which drive it well outside the allowed range. However, it should be noted that this problem arises in the simplest realization of this scheme. In complete models correctly incorporating fermion masses and mixing,  $m_b$  can receive extra corrections which may make it compatible with experiment. Also, small GUT threshold corrections to gauge coupling unification can help to reduce  $m_b$ . (For a brief discussion of the possibilities to remedy the  $m_b$  problem encountered in

the  $\mu < 0$  case and some relevant references see Ref. [5].) So, we do not consider this  $b$ -quark mass problem absolutely fatal for the  $\mu < 0$  case.

Be that as it may, it is certainly interesting to examine the alternative scenario with  $\mu > 0$  too. The  $b$ -quark mass receives negative SUSY corrections and can easily be compatible with data in this case. This scheme, however, is severely restricted by the recent experimental results [6] on the inclusive decay  $b \rightarrow s\gamma$  [7]. It is well-known that the SUSY corrections to the inclusive branching ratio  $\text{BR}(b \rightarrow s\gamma)$ , in the case of the MSSM with universal boundary conditions, arise mainly from chargino loops and have the same sign with the parameter  $\mu$ . Consequently, these corrections interfere constructively with the contribution from the standard model (SM) including an extra electroweak Higgs doublet. However, this contribution is already bigger than the experimental upper bound on  $\text{BR}(b \rightarrow s\gamma)$  for not too large values of the CP-odd Higgs boson mass  $m_A$ . As a result, in the present context with Yukawa unification and hence large  $\tan\beta$ , a lower bound on  $m_A$  is obtained [8,9] for  $\mu > 0$ . On the contrary, for  $\mu < 0$ , the SUSY corrections to  $\text{BR}(b \rightarrow s\gamma)$  interfere destructively with the SM plus extra Higgs doublet contribution yielding, in most cases, no restrictions on the parameters.

An additional constraint results from the requirement that the relic abundance  $\Omega_{LSP} h^2$  of the lightest supersymmetric particle (LSP) in the universe does not exceed the upper limit on the cold dark matter (CDM) abundance implied by cosmological considerations ( $\Omega_{LSP}$  is the present energy density of the LSPs over the critical energy density of the universe and  $h$  is the present value of the Hubble constant in units of  $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ ). Taking both the currently available cosmological models with zero/nonzero cosmological constant, which provide the best fits to all the data, as equally plausible alternatives for the composition of the energy density of the universe and accounting for the observational uncertainties, we obtain the restriction  $\Omega_{LSP} h^2 \lesssim 0.22$  (see Refs. [5,10]). Assuming that all the CDM in the universe is composed of LSPs, we further get  $\Omega_{LSP} h^2 \gtrsim 0.09$ .

The LSP is normally the lightest neutralino ( $\tilde{\chi}$ ). In the particular case of Yukawa unification, this neutralino turns out to be an almost pure bino. Its relic abundance has been estimated, for  $\mu > 0$ , and shown [8,9] to be well above unity, thereby overclosing the universe for all  $m_A$ 's permitted by  $b \rightarrow s\gamma$ . So, the combination of CDM considerations

and the data on  $\text{BR}(b \rightarrow s\gamma)$  seems to rule out the MSSM with  $\mu > 0$ , Yukawa unification and radiative electroweak breaking with universal boundary conditions.

It is important to note that, in Refs. [8,9], the coannihilation [11] of the LSP with the next-to-lightest supersymmetric particle (NLSP) has been ignored and only the LSP annihilation processes have been taken into account. It is well-known, however, that these coannihilations can be extremely important, if the mass of the NLSP is relatively close to the mass of the LSP, resulting to a considerable reduction of the LSP relic abundance [5,12,13]. The question then arises whether, by employing LSP-NLSP coannihilation, one can succeed reducing  $\Omega_{LSP} h^2$  below 0.22 for some values of  $m_A$  allowed by  $b \rightarrow s\gamma$  in the  $\mu > 0$  case. This would revitalize a part of the available parameter space for  $\mu > 0$  saving the simple, elegant and predictive MSSM with universal boundary conditions and Yukawa unification even in its simplest realization (with no need of extra corrections to  $m_b$ ). Although, in this parameter range, the sparticles would be quite massive (due to the lower bound on  $m_A$  from  $b \rightarrow s\gamma$ ) to be of immediate phenomenological interest, we would consider this as a very positive development.

In this paper, we reconsider the constraints from  $m_b$ ,  $b \rightarrow s\gamma$  and the LSP relic density in the context of MSSM with universal boundary conditions and Yukawa unification by incorporating the LSP ( $\tilde{\chi}$ ) and NLSP coannihilation in the calculation of  $\Omega_{LSP} h^2$ . The NLSP turns out to be the lightest stau mass eigenstate  $\tilde{\tau}_2$  and its coannihilation with bino has been studied in Ref. [5] (or [13]) for large (or small)  $\tan\beta$ . Although our analysis covers both signs of  $\mu$ , our main interest here is to see whether, for  $\mu > 0$ , there exists a range of parameters where all these constraints are simultaneously satisfied.

We will consider the MSSM with Yukawa unification described in detail in Ref. [5] and closely follow the notation as well as the renormalization group (RG) and radiative electroweak symmetry breaking analysis of this reference. The only essential improvement here is the inclusion of the full one-loop radiative corrections to the effective potential for the electroweak symmetry breaking which have been evaluated in Ref. [14] (Appendix E). We also incorporate the one-loop corrections to certain particle masses from the same reference and the two-loop corrections to the CP-even neutral Higgs boson masses (see below). In Ref. [5], we used a constant common SUSY threshold  $M_S = 1$  TeV, where the RG-improved tree-level potential was minimized and the parameters  $\mu$  and

$m_A$  were evaluated. Also, the MSSM RG equations were replaced by the SM ones below the same scale  $M_S$ . Here, we use a variable common SUSY threshold  $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ , where  $\tilde{t}_{1,2}$  are the two stop mass eigenstates. As it turns out, this does not make a very significant numerical difference since the values of  $\mu$  and  $m_A$  are found to be pretty stable to variations of  $M_S$ . Despite this fact, it is more appropriate to use a variable SUSY threshold here because of the wide variation of the sparticle spectrum encountered. This variation appears since, in this work, we consider not only relatively small but also quite large values of  $m_A$ . Finally, note that our choice of the SUSY threshold  $M_S$  minimizes the size of the one-loop corrections to  $\mu$  and  $m_A$  and, thus, the errors in the determination of these parameters.

We take universal soft SUSY breaking terms at  $M_{GUT}$ , i.e., a common mass for all scalar fields  $m_0$ , a common gaugino mass  $M_{1/2}$  and a common trilinear scalar coupling  $A_0$ , which we put equal to zero (we will discuss later the influence of non-zero  $A_0$ 's). Our effective theory below  $M_{GUT}$  then depends on the parameters ( $\mu_0 = \mu(M_{GUT})$ )

$$m_0, M_{1/2}, \mu_0, \alpha_G, M_{GUT}, h_0, \tan \beta ,$$

where  $\alpha_G = g_G^2/4\pi$  ( $g_G$  being the GUT gauge coupling constant) and  $h_0$  is the common top, bottom and tau Yukawa coupling constant at  $M_{GUT}$ . The values of  $\alpha_G$  and  $M_{GUT}$  are obtained as described in Ref. [5].

It was pointed out [3] that, for every  $m_A$ , the requirement of successful radiative electroweak symmetry breaking with Yukawa unification implies a relation among the GUT parameters  $m_0$  and  $M_{1/2}$ . This relation combined with any given ratio of the masses of the two lightest SUSY particles (lightest neutralino and stau) leads [5] to a complete determination of the values of  $m_0$  and  $M_{1/2}$  and hence the whole SUSY spectrum for every value of  $m_A$  (see also Ref. [15]). We will thus use  $m_A$  and  $\Delta_{\tilde{\tau}_2} = (m_{\tilde{\tau}_2} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$  as our basic independent parameters. For reasons to become obvious later, we concentrate here on the limiting case with  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$  ( $\Delta_{\tilde{\tau}_2} = 0$ ) where the LSP and NLSP coannihilation is [5] most efficient. The values of  $m_0$  and  $M_{1/2}$  can then be found as functions of  $m_A$  and are depicted in Fig.1, for  $\mu > 0$ , together with the LSP mass ( $m_{\tilde{\chi}}$ ) and the SUSY threshold mass parameter  $M_S$ . Note that these mass parameters are affected very little by changing the sign of  $\mu$ .

We find that the effect of including the full one-loop effective scalar potential from Ref. [14] is not very significant. In particular, the correction to the tree-level value of  $m_A$  is found to range from about  $-7.5\%$  to about  $1.5\%$  for  $m_A$  between 100 and 700 GeV with only a small dependence on the sign of  $\mu$ . Also, the one-loop radiative correction to the tree-level value of  $\mu$  turns out to be of the same sign as  $\mu$  and less than about  $3.5\%$  in the same range of  $m_A$ .

The values of the unified Yukawa coupling constant  $h_0$  at  $M_{GUT}$  and of  $\tan\beta$  at  $M_S$  are estimated by using the running top quark mass at  $m_t$ ,  $m_t(m_t) = 166$  GeV, and the running tau lepton mass at  $m_Z$ ,  $m_\tau(m_Z) = 1.746$  GeV. We also incorporate the SUSY threshold correction to  $m_\tau(M_S)$  from the approximate formula of Ref. [14]. This correction arises mainly from chargino/tau sneutrino loops, is almost  $m_A$ -independent and has the same sign as  $\mu$ . It is about  $8\%$ , for  $\mu > 0$ , leading to a value of  $\tan\beta = 55.4 - 54.5$  for  $m_A = 100 - 700$  GeV, while, for  $\mu < 0$ , we find a correction of about  $-7\%$  and  $\tan\beta = 47.8 - 46.9$  in the same range of  $m_A$ .

The tree-level values which we find for  $m_b(m_Z)$  are quite close to its experimental upper bound [16]:

$$m_b(m_Z) = 2.67 \pm 0.50 \text{ GeV}.$$

The SUSY correction [3,4] to the bottom quark mass is known to be very large for models with Yukawa unification. This correction originates mainly from squark/gluino and squark/chargino loops and has sign opposite to the one of  $\mu$  (in our convention). Thus, for  $\mu < 0$ , the corrected  $m_b(m_Z)$  will certainly be outside the experimentally allowed range. For  $\mu > 0$ , however, this large negative correction may easily make  $m_b(m_Z)$  compatible with data. Indeed, using the approximate formula of Ref. [14], we find that, for  $\mu < 0$ , the correction is about  $27.2\% - 23.8\%$  for  $m_A = 100 - 700$  GeV, which added to a tree-level value of  $m_b(m_Z) \approx 3.41$  GeV leads to an unacceptably large  $m_b(m_Z)$ . For  $\mu > 0$ , the tree-level value of  $m_b(m_Z)$  is equal to about 3.13 GeV and the SUSY correction, to be subtracted from it, about  $26.6\% - 24.7\%$  for  $m_A = 100 - 700$  GeV. The resulting bottom quark mass is then perfectly acceptable in this case. Note that the variation of the tree-level value of  $m_b(m_Z)$  with the sign of  $\mu$  is due to the SUSY corrections to  $m_\tau$  considered in calculating  $\tan\beta$ .

We incorporate in our calculation the two-loop corrections to the CP-even neutral Higgs boson mass matrix by employing the program *FeynHiggsFast* [17]. The tree-level mass of the lightest neutral CP-even Higgs boson is very close to  $m_Z$  for all  $m_A$ 's considered here. Two-loop corrections, however, increase its value,  $m_h$ , considerably. The lower experimental bound on  $m_h$ , say equal to 105 GeV, corresponds to  $m_A \approx 104$  GeV. The value of  $m_h$  increases rapidly with  $m_A$  reaching about 115 GeV at  $m_A = 120$  GeV. After this, the growth of  $m_h$  slows down drastically and this mass soon enters into a plateau with  $m_h \approx 122$  GeV. The difference between the two-loop and the tree-level value of the heavier CP-even neutral Higgs boson mass  $m_H$  is insignificant for values of  $m_A$  bigger than about 120 GeV. For smaller  $m_A$ 's, this difference increases reaching about 10% of the tree-level value of  $m_H$  at  $m_A \approx 100$  GeV. Finally, we also include the one-loop corrections to the charged Higgs boson mass using the formalism described in the appendices of Ref. [14] but without the neutralino and chargino contributions since, as it turns out, these contributions are relatively unstable as we change the scale  $M_S$ . Presumably, higher order corrections will alleviate this instability. It is, certainly, reassuring for our procedure that similar results are obtained by using the approximate formula of Ref. [18]. We find that one-loop corrections increase the tree-level value of the charged Higgs boson mass  $m_{H^\pm}$ . This increase ranges from 7% to 5% of the tree-level value as  $m_A$  changes from 100 to 700 GeV.

To study the constraints imposed by  $b \rightarrow s\gamma$  on the parameter space of our model, we follow the analysis of Ref. [19]. We consider the SM contribution to the inclusive branching ratio  $\text{BR}(b \rightarrow s\gamma)$  from a loop with a  $W$ -boson and top quark ( $t$ ), the contribution from loops with charged Higgs bosons (with mass corrected to one-loop) and  $t$  and the dominant SUSY contribution arising from loops with charginos and stop quarks. The SM contribution, which is factorized out in the formalism of Ref. [19], includes the next-to-leading order (NLO) QCD [20] and the leading order (LO) QED [19,21] corrections. The NLO QCD corrections [22] to the charged Higgs boson contribution are taken from the first paper in Ref. [22]. The SUSY contribution is evaluated by including only the LO QCQ corrections [7,23] using the formulae in Ref. [23]. NLO QCD corrections to the SUSY contribution have also been discussed in Ref. [23], but only under certain very restrictive conditions which never hold in our case since the lightest stop quark mass is

comparable to the masses of the other squarks and the gluinos. Also, the charginos are pretty heavy. We, thus, do not include these corrections in our calculation. (Moreover, as pointed out in Ref. [24], the available NLO QCD corrections to the SUSY contribution are not applicable to models with large values of  $\tan\beta$ , which is our case here.) The results, evaluated with central values of the input parameters and the renormalization and matching scales, are depicted in Fig.2 for both signs of  $\mu$ ,  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$  (see below). The charged Higgs contribution, added to the one of the SM, raises the predicted value of  $\text{BR}(b \rightarrow s\gamma)$  above the experimental upper bound  $4.5 \times 10^{-4}$  [6] for not too large values of  $m_A$ . The SUSY contribution, which becomes less important as  $m_A$  increases, interferes constructively or destructively with the other two contributions for  $\mu$  positive or negative respectively.

We see from Fig.2 that the SM plus charged Higgs contribution decreases as  $m_A$  (or  $m_{H^+}$ ) increases and enters into the experimentally allowed range at  $m_A \approx 295$  GeV corresponding to  $m_{H^+} \approx 318$  GeV and  $M_S \approx 1851$  GeV. For  $\mu < 0$ , inclusion of the SUSY contribution makes the  $\text{BR}(b \rightarrow s\gamma)$  compatible with data for all values of  $m_A$  explored here and no useful restrictions on the parameter space are obtained for  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$  and  $A_0 = 0$ . (We may, though, obtain restrictions for  $m_{\tilde{\tau}_2}$ 's higher than  $m_{\tilde{\chi}}$  and/or  $A_0 \neq 0$ .) For  $\mu > 0$ , however, the SUSY contribution increases the discrepancy between the predicted value of this branching ratio and the data. The upper experimental limit on  $\text{BR}(b \rightarrow s\gamma)$  ( $\approx 4.5 \times 10^{-4}$ ) is reached at  $m_A \approx 385$  GeV corresponding to  $m_{\tilde{\chi}} \approx 694$  GeV,  $m_0 \approx 781$  GeV,  $M_{1/2} \approx 1512$  GeV and  $M_S \approx 2418$  GeV. We conclude that, for  $\mu > 0$ ,  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ ,  $m_A$  should be greater than about 385 GeV for satisfying the constraints from the  $b \rightarrow s\gamma$  process. It is important to observe that, for values of  $m_{\tilde{\tau}_2}$  higher than  $m_{\tilde{\chi}}$ , charginos become even heavier and the lower bound on  $m_A$  decreases slightly but it can never become smaller than the bound (295 GeV) from the SM plus charged Higgs contribution.

The lower bound on  $m_A$  found by using central values of the input parameters can be considerably reduced if the theoretical uncertainties entering into the calculation of  $\text{BR}(b \rightarrow s\gamma)$  are taken into account. These uncertainties originating from the experimental errors in the input parameters and the ambiguities in the renormalization and matching scales are known to be quite significant. The SM contribution alone, which is



factorized out, generates an uncertainty of about  $\pm 10\%$ . The charged Higgs and SUSY contributions can only increase this uncertainty. In particular, the SUSY prediction for  $\mu > 0$  should not be a line in Fig.2 but rather a band with the appropriate error margin. Consequently, it should intersect the upper experimental bound line at a segment rather than at a point. This segment is found to be from about 300 to 510 GeV if only the ambiguities from the SM contribution are taken into account. We see that the lower bound on  $m_A$  is reduced from about 385 to about 300 GeV at least. Inclusion of the errors from the charged Higgs and chargino contributions can reduce this bound even further. However, we believe that they cannot be reliably calculated at the moment since the NLO QCD corrections to the SUSY contribution are not known in our case. In any case, the lower bound of 300 GeV on  $m_A$  is more than adequate for our purpose here, which is to revitalize the MSSM with Yukawa unification, universal boundary conditions and  $\mu > 0$ . Note that, for values of  $m_{\tilde{\tau}_2}$  higher than  $m_{\tilde{\chi}}$  this lower bound on  $m_A$  decreases slightly but it can never become smaller than the corresponding bound ( $\approx 200$  GeV) derived from the SM plus charged Higgs contribution.

The relic abundance of the LSP ( $\tilde{\chi}$ ), which is a nearly pure bino in our model, can be calculated by employing the analysis of Ref. [5] which is appropriate for Yukawa unification and, thus, large  $\tan \beta$ . The inclusion of coannihilation effects of the LSP with the NLSP, which is the lightest stau mass eigenstate ( $\tilde{\tau}_2$ ), is of crucial importance. These effects can reduce considerably the LSP relic abundance  $\Omega_{LSP} h^2$  so as to have a chance to satisfy the upper bound 0.22, derived from CDM considerations, for values of  $m_A$  consistent with the constraints from  $b \rightarrow s\gamma$ . In order to achieve maximal coannihilation and thus obtain the strongest possible reduction of  $\Omega_{LSP} h^2$ , we consider the limiting case  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$  (see Ref. [5]). The relevant coannihilation processes and Feynman graphs together with the corresponding analytical expressions can be found in Ref. [5]. Our computation here is, however, more accurate since it includes the one-loop corrections to the parameter  $\mu$ , the  $\tau$ -lepton mass,  $m_A$ ,  $m_{H^\pm}$  and the two-loop corrections to the CP-even neutral Higgs boson masses. The results are shown in Fig.3 for  $\mu > 0$  and  $A_0 = 0$ . For comparison, we also include the value of  $\Omega_{LSP} h^2$  obtained by ignoring coannihilation effects. Note that these results remain essentially unaltered by changing the sign of  $\mu$ .

From Fig.3, one readily finds that the CDM constraint on the LSP relic density

$0.09 \lesssim \Omega_{LSP} h^2 \lesssim 0.22$  is satisfied for  $m_A$ 's between about 275 and 400 GeV for  $\mu$  positive,  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ . Of course, the upper bound on  $m_A$  is more general than the lower one since it holds even if the CDM does not solely consist of LSPs. Note that the upper bound on  $m_A$  ( $\approx 400$  GeV) in the  $\mu > 0$  case corresponds to  $m_{\tilde{\chi}} \approx 724$  GeV,  $m_0 \approx 815$  GeV,  $M_{1/2} \approx 1575$  GeV and  $M_S \approx 2513$  GeV.

It is obvious, from Fig.3, that the reduction of  $\Omega_{LSP} h^2$  caused by the coannihilation effects is dramatic and can bring the LSP relic abundance below 0.22 for  $m_A$ 's in the allowed range from  $b \rightarrow s\gamma$  considerations in the  $\mu > 0$  case with  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ . Indeed, at  $m_A \approx 300$  GeV, which is its lower bound if a 10% theoretical error is allowed in  $\text{BR}(b \rightarrow s\gamma)$ ,  $\Omega_{LSP} h^2 \approx 0.112$  (or 3.92 with no coannihilation) and increases with  $m_A$  reaching 0.22 (or 7.4 with no coannihilation) at  $m_A \approx 400$  GeV, where (90% of) the central value of  $\text{BR}(b \rightarrow s\gamma)$  is about  $(4 \times 10^{-4})$   $4.44 \times 10^{-4}$ . We see that, for  $\Delta_{\tilde{\tau}_2} = 0$ ,  $A_0 = 0$  and  $\mu > 0$ , there exists a range of  $m_A$  (at least between 300 and 400 GeV) where both the constraints from  $b \rightarrow s\gamma$  and CDM can be satisfied. Note that, even with the central value of  $\text{BR}(b \rightarrow s\gamma)$ , this range does not disappear. It only shrinks to the interval between 385 and 400 GeV. The value of  $\Omega_{LSP} h^2$  with (without) coannihilation at  $m_A \approx 385$  GeV is about 0.205 (6.85).

So far we concentrated in the limiting case  $\Delta_{\tilde{\tau}_2} = 0$  where the coannihilation effects are more efficient and we took, for simplicity,  $A_0 = 0$ . We will now briefly discuss the effect of allowing general values of these quantities. Obviously, for any given  $m_A$ , the sparticles become heavier as we increase  $\Delta_{\tilde{\tau}_2}$ . The main effect of this is that coannihilation quickly fades away and the upper bound on  $m_A$  from CDM considerations rapidly decreases. As a consequence, there exists an upper bound on the parameter  $\Delta_{\tilde{\tau}_2}$  beyond which the allowed range of  $m_A$  disappears. Positive values of  $A_0$  lead to heavier sparticle masses and, thus, to an increase of  $\Omega_{LSP} h^2$  and a slight decrease of  $\text{BR}(b \rightarrow s\gamma)$ . The allowed range of  $m_A$  again disappears above a positive value of  $A_0$ . Negative  $A_0$ 's bigger than about  $-0.5M_{1/2}$  (generally) produce an insignificant decrease in the sparticle masses and  $\Omega_{LSP} h^2$ . Lower negative values of  $A_0$ , however, lead again to an increase of the sparticle masses and  $\Omega_{LSP} h^2$ , which means that a negative lower bound on  $A_0$  must also exist.

We will not undertake here the difficult task of constructing the region in the  $m_A$ ,  $\Delta_{\tilde{\tau}_2}$ ,  $A_0$  space which is consistent with the constraints from  $b \rightarrow s\gamma$  and CDM consid-

erations. This would require not only a detailed study of the theoretical uncertainties in the calculation of  $\text{BR}(b \rightarrow s\gamma)$ , but also inclusion of the uncertainties associated with the particular implementation of the radiative electroweak symmetry breaking, the RG analysis and the radiative corrections to various particle masses. These uncertainties propagated to the sparticle spectrum,  $\Omega_{LSP} h^2$  and  $\text{BR}(b \rightarrow s\gamma)$  can only widen the allowed region in the parameter space. Our main conclusion, which is the viability of MSSM with Yukawa unification, universal boundary conditions and  $\mu > 0$ , can only be further strengthened by including these errors.

In summary, we have considered the MSSM based on radiative electroweak symmetry breaking with universal boundary conditions and assumed unification of all three third family Yukawa couplings at the GUT scale. We employed the full one-loop effective potential for electroweak symmetry breaking as well as the one-loop corrections to the charged Higgs boson,  $b$ -quark and  $\tau$ -lepton masses. Also, two-loop corrections to the CP-even Higgs boson masses were taken. We imposed the constraints from  $b \rightarrow s\gamma$  and CDM in the universe by carefully including in the LSP relic abundance calculation the LSP (bino) and NLSP (lightest stau) coannihilation effects in the case of Yukawa unification (and, thus, large  $\tan\beta$ ). The calculation of the branching ratio of  $b \rightarrow s\gamma$  incorporates all the applicable NLO QCD and LO QED corrections and some of its theoretical errors were taken into account. We found that bino-stau coannihilation drastically reduces the LSP relic density and succeeds to bring it below the CDM upper bound for  $m_A$ 's which are allowed by  $b \rightarrow s\gamma$  in the  $\mu > 0$  case. This, combined with the fact that, for  $\mu > 0$ , the bottom quark mass after SUSY corrections is experimentally acceptable, shows that the simple, elegant and restrictive version of MSSM with Yukawa unification and universal boundary conditions can be perfectly viable. It is important to note that, without bino-stau coannihilation, this model was excluded.

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# FIGURES

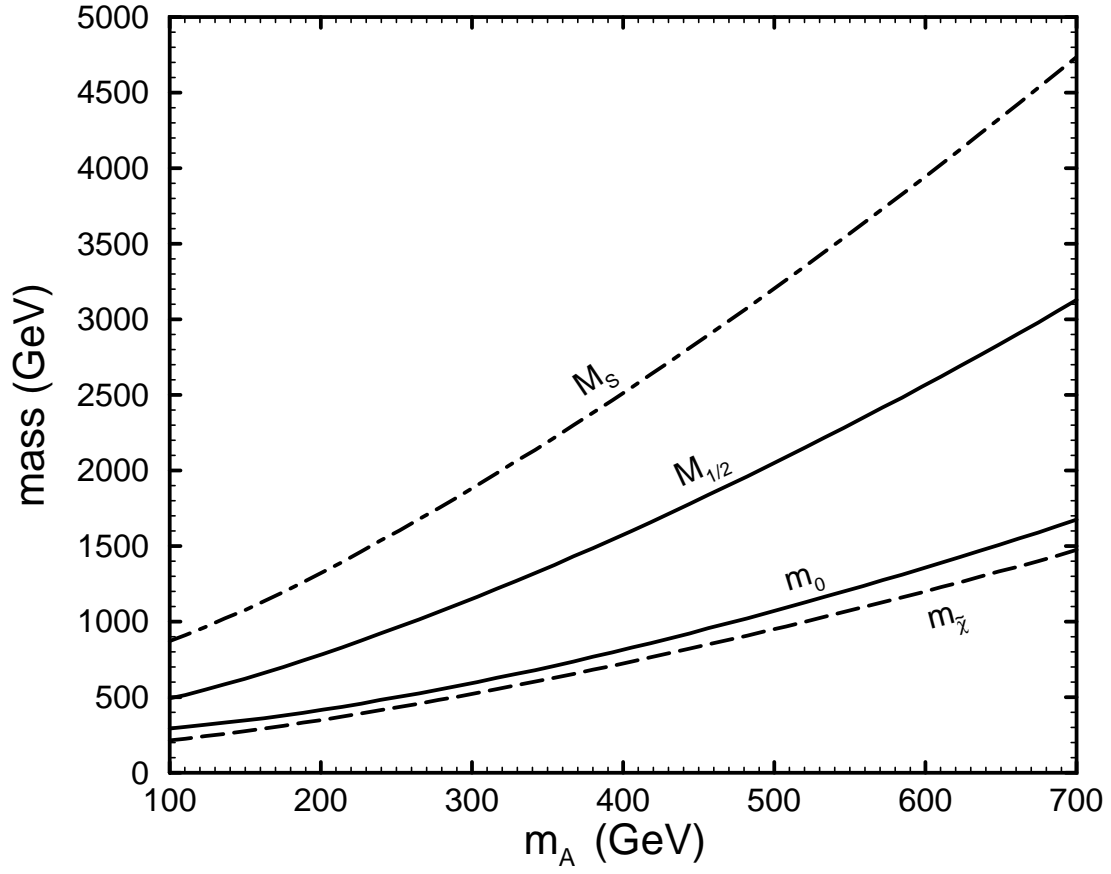


FIG. 1. The values of  $m_{\tilde{\chi}}$ ,  $m_0$ ,  $M_{1/2}$  and  $M_S$  as functions of  $m_A$  for  $\mu > 0$ ,  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ . These values are affected very little by changing the sign of  $\mu$ .

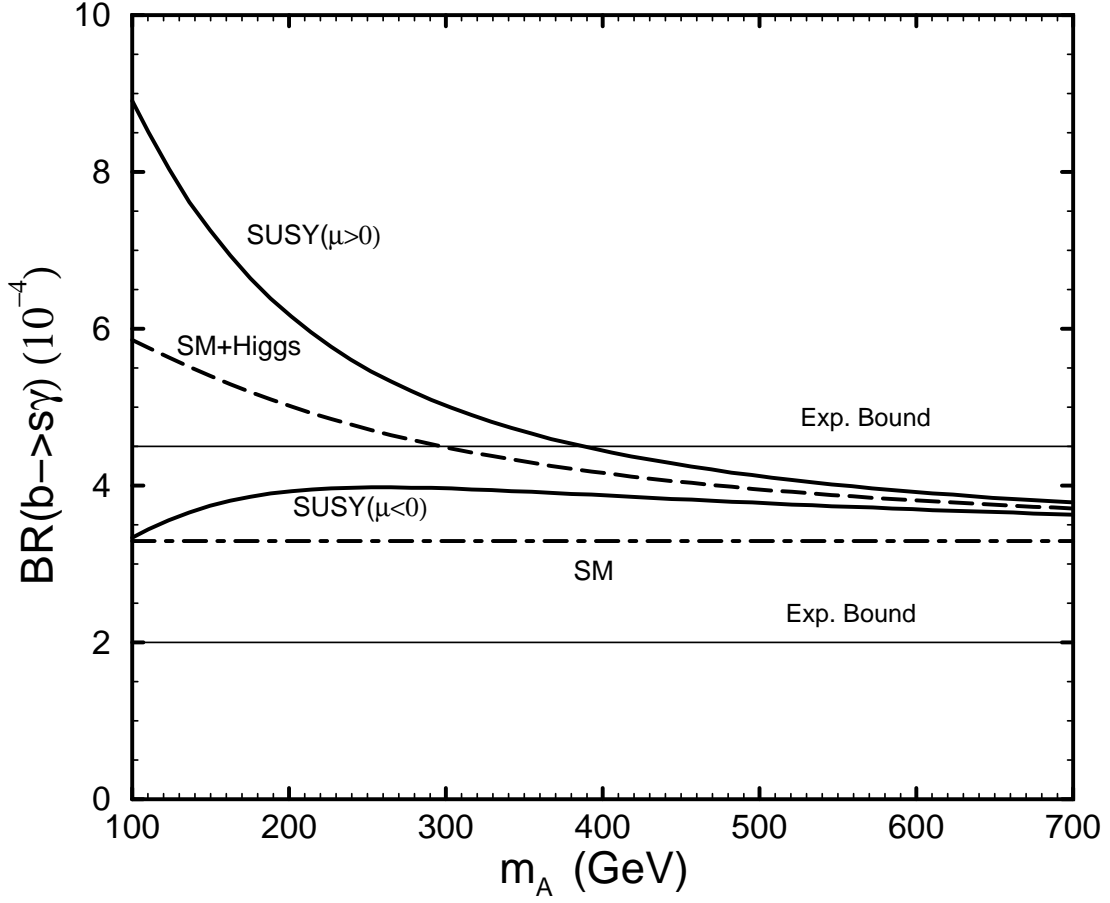


FIG. 2. The central value of the SUSY inclusive  $\text{BR}(b \rightarrow s\gamma)$  as function of  $m_A$  for both signs of  $\mu$ ,  $A_0 = 0$  and  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ . The contributions from the SM and the SM plus charged Higgs boson (SM+Higgs) as well as the experimental bounds on  $\text{BR}(b \rightarrow s\gamma)$ ,  $2 \times 10^{-4}$  and  $4.5 \times 10^{-4}$ , are also indicated.

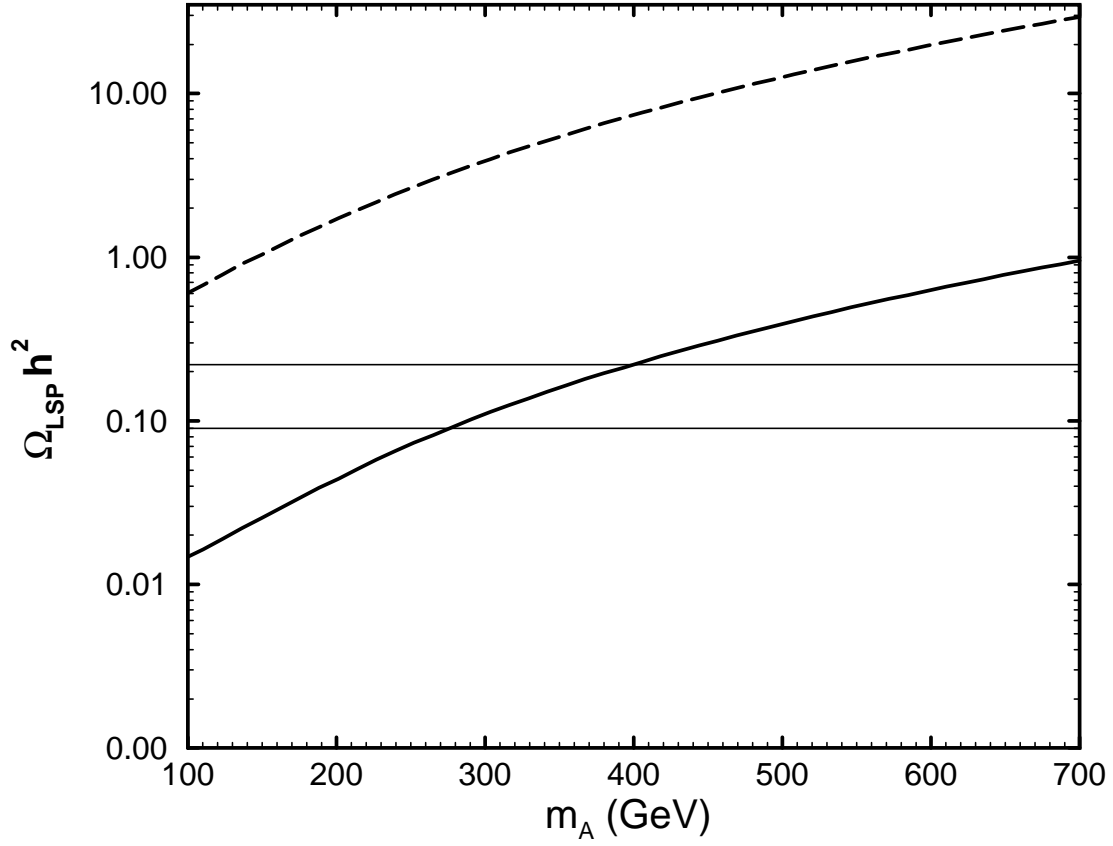


FIG. 3. The LSP relic abundance  $\Omega_{LSP} h^2$  as function of  $m_A$  in the limiting case  $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$  and for  $\mu > 0$ ,  $A_0 = 0$ . The solid line includes coannihilation of  $\tilde{\tau}_2$  and  $\tilde{\chi}$ , while the dashed line is obtained by only considering the LSP annihilation processes. These results are affected very little by changing the sign of  $\mu$ . The limiting lines at  $\Omega_{LSP} h^2 = 0.09$  and  $0.22$  are also included.